NPTEL WEB COURSE
SPACE FLIGHT MECHANICS

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Questions

1. A particle moves in a circle for the case of centre of force lying on the circumference. Derive the law of force underlying this motion. Assume the necessary variables.

2. Show that in elliptic motion about a focus under attraction $\frac{\mu}{r^2}$, the radial velocity is given by the equation $\dot{r} = \sqrt{\frac{\mu}{r^2}\left[a(1+e)-r\right]\left[r-a(1-e)\right]}$, where r is the radius, a is the semi-major axis, e is the eccentricity, and μ is the planetary gravitational constant.

3. Prove that the earth takes two days more than half a year to travel over half of its orbit separated by minor axis away from the sun. It is given that period of the earth is 365 days and the eccentricity of the orbit is 1/60.

4. A wheel of radius 1 m is rolling without slip on a belt which is moving to the left at a speed on 2 m/s to the left. An observer on the ground sees that the center of the wheel is moving to the right at a speed of 4 m/s. Determine the velocity of the point Q with respect to the ground and the belt.

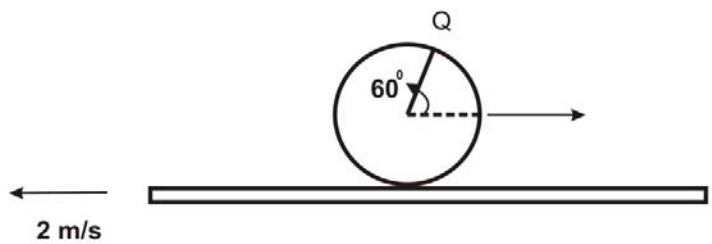

Figure 1: Wheel rolling on the belt

5. Find an equation for the speed V of a satellite in the earth orbit as a function of energy E and the radial distance r.

6. Of two circular orbits, the one with larger radius has greater energy E, explain the reason for lower velocity in this orbit as compared to that with the smaller radius.

7. Show that central force motion is confined to a fixed plane.

8. Derive an expression for eccentricity e in terms of an initial speed V_0, and flight path angle ϕ_0.

9. Show that the speed of a satellite in an elliptic orbit at the either end of the minor axis is the same as circular speed at that point.

10. Prove that the flight path angle ϕ is 45^0 when $\theta=90^0$ for all parabolic trajectories.

11. Determine the orbital period of a spacecraft whose perigee and apogee are at an altitude of 122 km and 622 km, respectively.

12. Show that the period of an orbit close to the surface of a homogeneous spherical planet is a function of planetary density only.

13. A small satellite of a certain planet moves along a spiral path described by rθ=C, where C is a constant. Derive the associated force law for this motion.

14. Halley's comet orbits the sun in an elliptical orbit of eccentricity 0.9673. Compare the linear and angular velocities of this comet at perihelion and aphelion.

15. Find the perihelion distance of a comet which describes a parabolic orbit in the plane of ecliptic and remains the longest time within the earth's orbit which is assumed to be circular.

16. Prove that the mean anomal M and the true anomal θ in elliptic orbit are related by the equation

$$M = (1-e^2)^{3/2}\int_0^\theta \frac{d\theta}{(1+e\cos\theta)^2}$$

Hence prove that to the order of e^2 ($O(e^2)$)

$$M = \theta - 2e\sin\theta + \frac{3}{4}e^2\sin 2\theta,$$

$$\theta = M + 2e\sin M + \frac{5}{4}\sin 2M.$$

17. A spaceprobe is moving in an elliptical orbit of period T under the attraction of the sum whose mass can be taken to be M. The onboard rockest are fired to increase its orbital speed V instantaneously by ΔV. Prove that the resulting change in the period is given by

$$\Delta T = 3(2\pi GM)^{-2/3}T^{5/3}V\Delta V$$

18. A rocket exits the earth's atmosphere just before the thrust is terminated at an altitude of 640 km. The geocentric velocity of the rocket is 10.4 km/s. In what direction it must be travelling in order to achieve maximum distance from the center of earth. Calculate this distance. If the direction of travel of the rocket at the termination of thrust had made an angle 88^0 with the geometric radius vector of the rocket, then calculate the period's the rocket's orbit.

19. The semimajor axis of a satellite when injected into the orbit has a semi major axis of 1.0478 earth radii and a period of 90.54 min. Calculate the mass of the earth in the units of the sun's mass.

20. A rocket travelling in a circular orbit $r_i v_i^2 / \mu = 1$ is given an impulsive thrust normal to the orbit so that the resultant velocity vector makes an angle ϕ_0 outward from the tangent to the departing circular orbit. Determine the new orbit, specifying the perigee and apogee distances and the eccentricity. Determine θ_0 to perigee.

21. A satellite is launched with the following conditions: $\frac{r_0 v_0^2}{\mu} = 1, \phi_0 = 20^0, \text{and } \frac{r_0}{R} = 2.0$. (a) Determine the orbit parameters e, a/R, and establish the initial position with respect to perigee. (b) Now imagine that this satellite continue along this orbit to $\theta=150^0$,at which time the orbit is to be increased to a value of a/R =3.60 without rotating the apse line, determine the required increment in the velocity and its direction. ϕ_0 is the flight path angle. R is the radius of earth. Sketch the figure showing all the parameters. If there are multiple solutions then state the reason of selecting a particular solution.

22. The earth's mean heliocentric velocity is 29.78 km/s. Assume that meteors travel in parabolic heliocentric orbits. Show that the speed of approach of meteors towards earth lies between 12.3 and 71.9 km/s.

23. On January 10, 1963 the heliocentric ecliptic rectangular coordinates of position and velocity of an interplanetary probe were $x = 0.68, y = 0.52, z = 0.18$ and $\dot{x} = -2.2, \dot{y} = 28.1, \dot{z} = 2.6$ respectively, the distance being measured in units of the Earth's semi major axis, the velocity in km/s. Find elements of the probe's orbit. $\left(\mu = 1.32715 \times 10^{11}\, km^3/s^2, r_e = 149.5 \times 10^6\, km\right)$.

24. The Skylab space station was launched with a Saturn V booster such that at the burnout of the last stage it had V_0=9.25 km/s, r_0=7000 km, and $\beta_0=0^0$. (a) A circular was desired. Was this objective achieved? Explain the reason for your answer. If the orbit were not circular, find the eccentricity. (b) What is the radial distance at the point of closest approach? (c) Calculate the specific energy, semi-major axis, and specific angular momentum.

25. Define the potential U at a point in the gravitational field due to a particle of mass M. Use this definition to obtain the following:

    (a) U=-KM/r for a spherical body of mass M, where r is the distance of the point from the centre of the shell.

(b) $U = -\frac{1}{2}K\sum_{i=1}\sum_{j=1}\frac{m_i m_j}{r_{ij}}, \quad i \neq j,$ for a system of n bodies of masses $m_i, i = 1,2,3,......,n$ whose mutual radius vectors are given by $\overline{r}_{ij}$.

26. A system of n particles of masses m_i , i=1,2,3,….,n moves under the action of a law of gravitation such that the force of attraction between each pair of the particles is directly proportional to the product of their masses and directly proportional to the distance between them. Show that under such a law the orbit of any particle about any other particle is an ellipse and that the other particle is at the center of the ellipse and that the orbit of any particle with respect to the center of mass of the system is also an ellipse.

27. Show that for μ = 0.5, the curves are symmetric about y-axis also and that the points L_1,L_2 are reached simultaneously when constant C is continuously decreased starting with a very large initial value. Compute the position of the five Lagrange points in the case μ = 0.5.

28. Prove that the minimum value of $2U = x^2 + y^2 + 2\frac{(1-\mu)}{r_1} + 2\frac{\mu}{r^2}$ in the restricted coplanar 3 – body problem is 3 so that no zero velocity curves exists for C<3. What is the shape of the curves for C=3.

29. If the subscript B denotes the Bary center of the system, derive the following equations of motion:

$$\frac{d^2rB_2}{dt^2} = -K\left(\sum_{i=1}^{n} m_i\right)\frac{xB_2}{r^3B_2} + K\sum_{j=1, j\neq 2}^{n} m_j x_{j2}\left(\frac{1}{r^3B_2} - \frac{1}{r_{j2}^3}\right)$$

Where B_j is distance of the j^{th} particle from the Bary center, n is number f masses , and K is gravitational constant.

30. For an n – body system, prove that $\frac{1}{2}\frac{d^2}{dt^2}\sum_{i=1}^{n}(m_i r_i^2) = T + h = U = 2h,$ where $h = T - U,$ show from this that if h>0 then $\sum_{i=1}^{n}(m_i r_i^2) \to \infty$ as $t \to \infty$. Conclude that at least one distance r_k cannot remain bounded.

31. The equation of the zero velocity curves of the Earth-Moon system is 2U=C. Determine the largest values of C for which the Earth-Moon trip is possible.

32. Using the sign and symbol representation of n – body system, define the system moment of inertia of the system as $I = \sum_{i=1}^{n} m_i r_i^2$. Show that $\ddot{I} = 2(T + E) = 2U - 4E$. Use the result $\sum_{i=1}^{n} \overline{r}_i.\nabla_i U = -U$.

33. Show that for Sun-Earth-Satellite system, the radius defining the single sphere of influence around the earth is approximately given by $r = (m/M)^{2/5} r_p$, where m, M ate the masses of the planet and the sun respectively, and r_p is the radius of the Earth around the sun.

34. Show that for sun-earth-satellite system the radii of the outer and inner sphere of influence d around the planets are given by

$$|\in_p| = \left(\frac{m}{M}\right)\frac{1}{d^2}\left(1 - \frac{d^2}{(1+d)^2}\right) \quad outer$$
$$|\in_s| = \left(\frac{m}{M}\right)d^2\left(1 - \frac{d^2}{(1+d)^2}\right) \quad inner$$

Where $d = \frac{\rho}{r}$. To derive the desired results assume that the vehicle lies on the line planet-sun between the two massive bodies and apply the conditions: $|P_p| \cong \in_p |A_s|$ and $|P_p| \cong \in_s |A_p|$.

35. A satellite is to be placed into an orbit of radius r_f through the direct ascent. This is followed by its injection into a transfer ellipse along which it coasts until the final orbit is reached where the final impulse places the satellite into the desired orbit. Show that the final cutoff velocity attained during powered ascent is given by

$$V^2 = (g_0 R^2 / r_i)\left[\frac{2(r_f - r_i) r_f \sin^2\theta_i}{r_f \sin^2\theta - r_i \sin^2\theta}\right]$$

36. A satellite is to be placed from an elliptic orbit of eccentricity e_i and major axis $2a_i$ to an outer coplanar coaxial elliptic orbit with eccentricity e_f and major axis $2a_f$. For two such possible cotangential transfer find the eccentricities and also calculate ΔV for these transfer.

37. Two satellites A and B are in the same circular orbit of radius $a = n_0 R$ in the same plane but B is leading A by the angle ϕ_{12}^0 where R is the radius of the earth. If A fires a retrorocket in the tangential direction, show that in order for the two satellite to intercept after A has completed one revolution of its

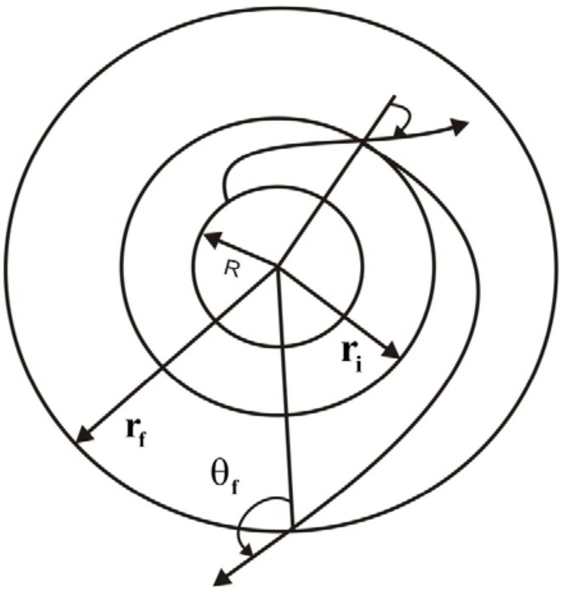

Figure 2: Ascent of rocket and injection into orbit

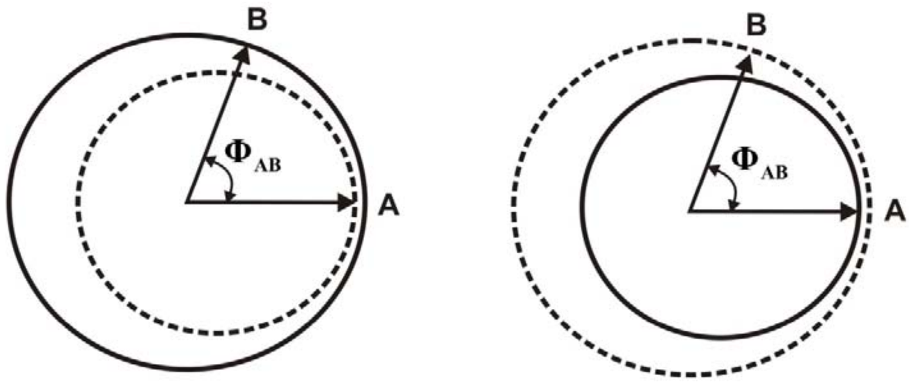

Figure 3: Two satellites in the same orbit

Circular orbit, the necessary increment in velocity is $\Delta V = V_c \left[1 - \sqrt{2 - \left(1 - \frac{\phi_{AB}^0}{360}\right)^{-2/3}} \right]$, where V_c is the circular orbit velocity. If the rocket is fired towards the rear so as to increase the velocity determine the V necessary to intercept vehicle on the N^{th} visit to A.

38. Satellite 2 is travelling east in equatorial circular orbit of $\frac{a}{R} = 2$, being at position longitude ϕ_2 =0, latitude λ_2=0 at time t=0. Satellite 1 at t=0 at the latitude λ_1=90^0 and traveling in polar elliptical orbit longitude ϕ_1 =0,with $\frac{a}{R} = 2$, e=0.30. If it is desired for 1 to intercept 2 at longitude 330^0, determine the transfer orbit and components of the velocity increment. The position of 1 at t=0 corresponds to perigee for elliptic orbit.

39. Two circular heliocentric orbits have radii 1 A.U. and 3 A.U. and a mutual inclination of 5^0. It is proposed to transfer a vehicle moving in the outer orbit by a single elliptic path into the inner one by applying the two velocity increments. When should they be applied? Should the change in orbit inclination be made at the outer or inner transfer points if a saving in fuel is to be made. Calculate the saving in the velocity sum if the correct decision is made.

40. It is proposed to study the sensitivity of transfer orbits to small errors in position and velocity cut-off. Obtain the expression for the resulting error in orbital elements.

41. The early warning defense system detects a Russian launched vehicle. If ground tracking station determine that at one point its flight parameters ground tracking stations determine that one point its flight

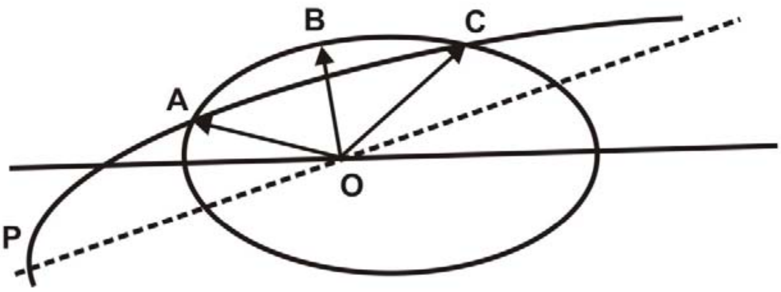

Figure 4: Rendezvous of satellites A and B

parameters are V=9 km/s , r = 7500 km, and flight path angle β=25^0, determine whether this is and ICBM, earth satellite, or solar probe. If it is an ICBM, calculate its range over the earth's surface otherwise determine the geocentric or heliocentric period.

42. A satellite is launched from earth such that the booster burnout occurs at an altitude of 422 km with a velocity V_0=9.5 km/s and flight path angle β_0 =10^0. (a) determine θ_0 and e, (b) determine a and r_p. (c) as the satellite passes perigee its rocket is fired in order to circularize the orbit. However, the resulting velocity was 4% off, and β=0.04 radian at the end of thrusting. What is the resulting value of ?

43. What is the minimum ΔV required to bring a synchronous satellite back to earth reentry altitude (about 75 km)?

44. A satellite transfers from a low circular orbit or radius 7000 km to a circular orbit with a period of 12 hours. (a) What is the radius of final orbit? (b) If a Hohmann transfer is used, calculate the eccentricity of the transfer ellipse and total velocity increment.(c) Is it possible to achieve fuel economy using belliptic 3 impulse transfer instead.

45. It is desired to tilt the orbital plane of a satellite by an angle ϕ without changing the perigee and apogee positions of its elliptic trajectory. This was achieved by applying the impulse at the perigee. (a) Find the required velocity increment in terms of ϕ and V where V is the velocity of the vehicle at the perigee. (b) We can achieve the tilt of the orbital plane by applying the impulse at the apogee. Which of the tow methods would consume less energy and why?

46. Two satellites have positions A and B at time t while moving in an elliptic orbit of eccentricity e about the center of force O. It is proposed t apply an impulse at A to send it in a faster trajectory of eccentricity e_t so that the satellites can be made to have a rendezvous when the satellite B moves to position C. Given that $\angle BOA = \phi_1$ and $\angle COB = \phi_2$ then show that

$$e_t = \frac{r_C - r_A}{r_A \cos\theta_A - r_C \cos(\theta_A + \phi_1 + \phi_2)}$$

Where r_x is the length of radius vector at point at point x=A,B,C, and θ_A is the true anomaly of A in the transfer orbit. Indicate the steps of the iteration schemes that would be required to determine a suitable orbit facilitating rendezvous.

47. The motion of a spinning rigid satellite in the absence of external torque is given by

$$I_{xx}\dot{\omega}_x = (I_{yy} - I_{zz})\omega_y\omega_z$$
$$I_{yy}\dot{\omega}_y = (I_{zz} - I_{xx})\omega_z\omega_x$$
$$I_{zz}\dot{\omega}_z = (I_{xx} - I_{yy})\omega_x\omega_y$$

Where x,y and z are the principal body coordinate axes with origin at the mass center of the rigid satellite, ω_x, ω_y and ω_z are the angular velocity components. Show that if rotation is given about x axis only, it would represent an equilibrium configuration.

48. Prove that the equation of a satellite pitching motion in a circular orbit is given by

$$\frac{d^2\psi}{d\theta^2} + 3k_i \sin\psi \cos\psi = 0$$

Find the approximate period of oscillations for motion in small. Integrate this equation to determine the first integral of pitching motion. Hence find the maximum pitching angle for the case when initial pitch angle is zero while initial pitchinh velocity (dimensionless) is $\sqrt{3/2}$ and k_i=1.

49. For the flight of a single stage rocket in vertical ascent, show that the rocket burnout altitude is given by

$$\frac{y_b - y_0}{g_0 I_{sp}^2} = \left(\frac{1}{r}\right)\left(1-\frac{1}{R}\right)\left[1-\ln\frac{R}{1-R}\right]-\frac{\left(1-\frac{1}{R}\right)^2}{2r^2}$$

50. A two stage rocket is to attain a maximum speed of 8 km/s with $I_{sp1} = I_{sp2} = 302\,s$ and the structural factor $\beta_i = \frac{m_{si}}{m_{pi} + m_{si}}$ is same for both the stages. Determine the mass ratio of each stage.