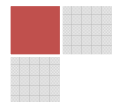


Module 1: Compressible Aerodynamics

Lecture 1: Compressible Aerodynamics



Thermodynamics

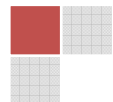
Experimental results are the basis of any physical theory. The experimental basis of thermodynamics is formalized in the principal laws. The law of conservation of energy is one of these principal laws. It introduces the concept of internal energy of a system. The other principal laws of thermodynamics introduce and define the properties and concepts of temperature and entropy. Classical thermodynamics is concerned, at any rate as the bulk of the subject stands, with equilibrium state of uniform matter, that is, with states in which all local mechanical, physical and thermal quantities are virtually independent of both position and time. Thermodynamical results may be applied directly to fluids at rest when their properties are uniform. A very little is known of the thermodynamics of non-equilibrium states. However, observation shows that results for equilibrium states are approximately valid for the non-equilibrium non-uniform states common in practical fluid dynamics; large through the departures from equilibrium in a moving fluid may appear to be, they are apparently small in their effect on thermodynamical relationships.

Fluid mechanics of perfect fluids (without viscosity and heat conductivity) is an extension of equilibrium thermodynamics to moving fluids. In addition to internal energy, kinetic energy of the fluid needs to be considered. The ratio of the kinetic energy per unit mass to the internal energy per unit mass is a characteristic dimensionless quantity of the flow problem and in the simplest cases is directly proportional to the square of Mach number.

In fluid mechanics of low speed flow, thermodynamic considerations are not needed: the heat content of the fluid is then so large compared to the kinetic energy of the flow that the temperature remains nearly constant even if the whole kinetic energy is transformed into heat. The opposite can be true in high-speed flow problems.

Thermodynamic system – a quantity of matter separated from the ‘surroundings’ or the ‘environment’ by an ‘enclosure’

Enclosure = a closed surface with its properties defined everywhere – may or may not transmit heat, work or mass.



The concepts of thermodynamics are helpful in fluid mechanics for the additional reason that in both subjects the objective is a set of results which apply to matter as generally as possible, without regard for the different molecular properties and mechanism at work. Additional results may be obtained by taking into account any known molecular properties of a fluid (with the aid of kinetic theory in case of certain gases).

It is a fact of experience that the state of a given mass of fluid in equilibrium (spatial and temporal uniformity) under the simplest possible conditions is specified uniquely by two parameters, which for convenience may be chosen as the specific volume $v = 1/\rho$ and the pressure p . All other quantities describing the state of the fluid are function of these two parameters of state. One of the most important of these quantities is the temperature. 'A mass of fluid in equilibrium has the same temperature as a test mass of fluid also in equilibrium if the two masses remain in equilibrium when placed in thermal contact' (Zeroth law). The relation between the temperature T and the two parameters of state may be written as

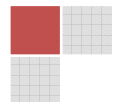
$$f(p, v, T) = 0$$

This exhibits formally the arbitrariness of the choice of the two parameters of state. The equation is called 'the equation of state'. Generally written as $p = p(v, T)$ and is called 'thermal equation of state'. Another important quantity describing the state of the fluid is the internal energy per unit mass e . The change in the internal energy of the system (mass of fluid) at rest consequent on a change of state is defined by the first law of thermodynamics, as being such as to satisfy the conservation of energy when account is taken of both heat given to the fluid and work done on the fluid. Thus if the state of a given uniform mass of fluid is changed by a gain of heat of amount Q per unit mass and by the performance of work on the fluid of amount W per unit mass, then

$$\Delta e = Q + W$$

$$e = e(v, T) \text{ is the caloric equation of state}$$

The internal energy e is a function of the state parameters, and the change which may be either infinitesimal or finite depends only on the initial and final states, but Q and W are measures of external effects and may separately depend on the particular way in which the transition between the two states is made. If the mass of fluid is thermally isolated from its surrounding so that no exchange of heat can occur, $Q = 0$, and the change of state of the fluid is 'adiabatic'.



The most important way of performing work on the system is compression. Analytic expression can be obtained if the change is 'reversible'. This implies that the change is carried out so slowly that fluid passes through a succession of equilibrium states, the direction of the change being without effect. At each stage, the pressure is uniform ' p ', so the work done on unit mass of fluid for small decrease in volume is $p\delta v$. Thus for a reversible transition from one state to another neighbouring state,

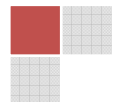
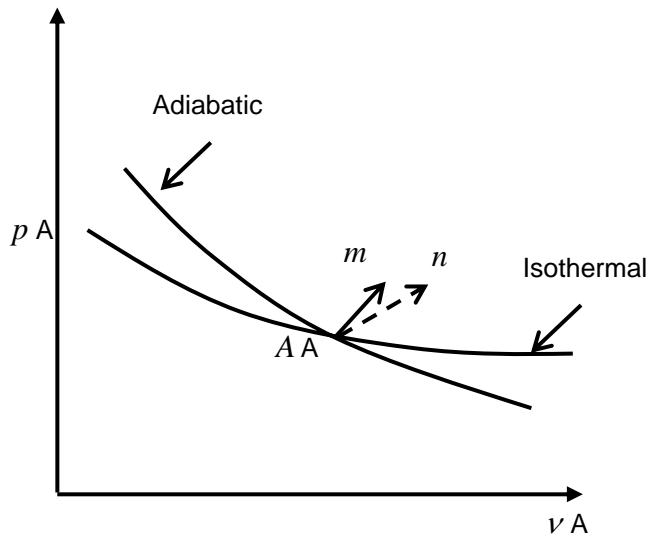
$$\delta e = \delta q - p\delta v$$

The particular path by which the initial and final equilibrium states are joined is relevant here, because p is not in general a function of v alone.

Another practical quantity of some importance is the 'specific heat' of the fluid, which is the amount of heat given to unit mass of the fluid per unit rise in temperature in a small reversible change. The specific heat may be written as

$$c = \frac{\delta Q}{\delta T}$$

This is not uniquely determined until the conditions under which the reversible changes occur are specified: An equilibrium state is a point on a (p, v) plane (indicator diagram) and a small reversible change $(\delta p, \delta v)$ starting from a point A may proceed in any direction.



If the only work done on the fluid is that done by compression, the heat δQ which must be supplied to unit mass is determined as

$$\delta Q = \left(\frac{\partial E}{\partial p} \right)_v \delta p + \left(\frac{\partial E}{\partial v} \right)_p \delta v + p \delta v$$

and the change in temperatures is

$$\delta T = \left(\frac{\partial T}{\partial p} \right)_v \delta p + \left(\frac{\partial T}{\partial v} \right)_p \delta v$$

The specific heat thus depends on the ratio $\delta p / \delta v$, and hence on the choice of direction of the change from A. Two particular well defined choices are changes parallel to the axes of the indicator diagram giving the principal specific heats

$$C_p = \left(\frac{\delta Q}{\delta T} \right)_p = \left(\frac{\partial E}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial T} \right)_p; \quad C_v = \left(\frac{\delta Q}{\delta T} \right)_v = \left(\frac{\partial E}{\partial T} \right)_v$$

δT varies sinusoidally as the point representing the final state moves round a circle of small radius centered on A, being zero on the isotherm through A and maximum in a direction \vec{m} normal to the isotherm. Likewise, δQ varies sinusoidally, being zero on the adiabat through A and a maximum in a direction \vec{n} normal to it. The components of the unit vectors being (m_v, m_p) and (n_v, n_p)

$$C_p = \frac{n_v (\delta Q)_{\max}}{m_v (\delta T)_{\max}}, \quad C_v = \frac{n_p (\delta Q)_{\max}}{m_p (\delta T)_{\max}}$$

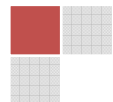
Since $-m_v/m_p$ and $-n_v/n_p$ are the gradients of the isothermal and adiabatic lines, the ratio of the principal specific heats is

$$\gamma = \frac{C_p}{C_v} = \left(\frac{n_v}{n_p} \right) / \left(\frac{m_v}{m_p} \right) = \left(\frac{\partial p}{\partial v} \right)_{adiab} / \left(\frac{\partial p}{\partial v} \right)_T \text{ or } = \left(\frac{\partial v}{\partial p} \right)_T / \left(\frac{\partial v}{\partial p} \right)_{ad}$$

Extensive variables: value depends on the mass of the system, Like M, E, V, S .

Intensive variables: variables that do not depend on the total mass of the system, like p, T .

Both sets refer to state variables only. For every extensive variable an intensive variable (per unit mass, or specific) can be introduced.



The weighted ratio $-v \delta p / \delta v - v \delta p / \delta v$ in a small reversible change is the 'bulk modulus of elasticity' of fluid. For fluid dynamical purposes, its reciprocal $-\frac{\delta v}{v \delta p}$ or $\frac{1}{\rho} \frac{\delta \rho}{\delta p}$ is more useful.

This is called the 'coefficient of compressibility'. Like 'specific heat', the 'bulk modulus' or the 'coefficient of compressibility' takes a different value for each direction of change. Adiabatic and isothermal changes correspond to two particular directions with special significance and the first law requires the ratio of the two corresponding bulk modulus to equal the ratio of the principal specific heats.

